Grade 7 Mathematics, Quarter 1, Unit 1.1

Applying and Extending the Understanding of Operations with Integers

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Describe and show real-world situations in which the sum of opposite quantities equals 0.
- Show that the distance between two numbers on a number line is the absolute value of their difference.
- Understand the relationship between addition and subtraction of integers.
- Interpret products and quotients in a real-world context.
- Apply properties of operations to add, subtract, multiply, and divide integers using appropriate strategies in real-world contexts.
- Understand the distributive property leading to rules for multiplying integers.
- Understand that integers can be divided, provided the divisor is not zero.
- Represent addition and subtraction of integers on a vertical and horizontal number line.

Mathematical practices to be integrated

Model with mathematics.

- Explain the meaning of rational numbers.
- Make conjectures about solutions to addition, subtraction, multiplication, and division of integer problems.
- Use number lines and/or algorithms to solve problems involving integers and all four mathematical operations.

Attend to precision.

- Ensure that the sign associated with the answer to their problem is appropriate based on mathematical models and algorithms used.

Look for and express regularity in repeated reasoning.

- Recognize that repeated calculations in addition can be simplified through multiplication.
- Recognize that division can simplify repeated calculations in subtraction.
- Use the distributive property for multiplication.
- Recognize the sum of a number and its opposite are zero (additive inverse).

Essential questions

- How would you explain the sum of (a) and ($-a$)?
- How can a number line help us find the distance between two integers?
- Why can't you have a divisor of zero?
- How are addition and subtraction of integers related?
- How does the distributive property of multiplication help you solve problems with integers?

- What strategies can be used to add and subtract integers?
- What strategies can be used to multiply and divide integers?
- How do you decide if your product or quotient makes sense?
- How do you use a number line to represent addition of integers?
- How do you use a number line to show subtraction of integers?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System **7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of ~~rational~~ numbers by describing real-world contexts.

c. Understand subtraction of ~~rational~~ numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two ~~rational~~ numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract ~~rational~~ numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from ~~fractions to rational~~ numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of ~~rational~~ numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a ~~rational~~ number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of ~~rational~~ numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide ~~rational~~ numbers.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students became fluent in operations with positive rational numbers in grade 6. They realized that opposites are equidistant from zero on vertical and horizontal number lines and can locate and position rational numbers on a coordinate plane. Students in sixth grade used coordinates and absolute value to find distance between points on a number line or coordinate plane.

Current Learning

Students add, subtract, multiply, and divide integers. Students are reinforcing operations with positive integers and developing operations with negative integers. They are expected to use distributive property, additive inverse, and absolute value to relate content to a real-world context. Students use prior knowledge of position on a number line to solve problems using the rules for operating with signed numbers. In a future unit, learners will expand operations to include all rational numbers.

Future Learning

These standards will support recognition of rational numbers versus irrational numbers in grade 8. They will also aid in students' ability to approximate irrational numbers.

Additional Findings

According to *Curriculum Focal Points,* "Students extend understanding of addition, subtraction, multiplication and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday context students explain why the rules for adding, subtracting, multiplying, dividing with negative numbers makes sense." (p. 19)

In *A Research Companion to Principles and Standards for School Mathematics*, page 68, the authors state, "Their development of computational fluency and their acquisition of problem-solving skills are intertwined as both develop with understanding." (p. 68)

According to *Principles and Standards for School Mathematics*, "Students understand the meanings of operations and how they relate to one another by learning the effects of arithmetic operations with fractions, decimals, and integers; using properties to simplify computations with integers, fractions, and decimals; and using the inverse relationships of addition and subtraction, and squaring and finding square roots to simplify computations and solve problems." (p. 214)

Grade 7 Mathematics, Quarter 1, Unit 1.2

Adding and Subtracting Rational Numbers

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Use the additive inverse to solve problems and recognize that the sum of opposites is zero.
- Interpret the sum of rational numbers by describing real-world context.
- Understand that the distance between two numbers on a number line is the absolute value of their difference.
- Apply properties of operations to add and subtract rational numbers using appropriate strategies in real-world contexts.
- Add rational numbers.
- Subtract rational numbers.
- Represent addition and subtraction of all rational numbers on a vertical and horizontal number line.

Mathematical practices to be integrated

Model with mathematics.

- Use number lines and/or algorithms to solve problems involving addition and subtraction of rational numbers.

Use appropriate tools strategically.

- Use number lines, chip model, and black and red pencils to solve problems involving addition and subtraction of rational numbers.

Attend to precision.

- Check answers to ensure that the sign associated with the answer to a problem is appropriate based on mathematical models and algorithms used.

Essential questions

- How would you explain the sum of $(1/a)$ and $(-1/a)$?
- How can a number line help us find the distance between two fractions if one is negative?
- How can a number line help us find the distance between two decimal numbers if one is a negative decimal number?
- How are addition and subtraction of rational numbers related?
- What strategies can be used to add rational numbers?
- What strategies can be used to subtract rational numbers?
- How can you describe a real-world context that shows the sum of a positive and negative fraction?
- How do you use a number line to represent addition of rational numbers?
- How do you use a number line to show subtraction of rational numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System	**7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grades 5 and 6, students gained fluency in adding, subtracting, multiplying, and dividing fractions with like and unlike denominators. They also developed a strategy for using equivalent fractions to add and subtract fractions. Students learned to divide multidigit positive numbers with decimals. They realized that opposites are equidistant from zero on vertical and horizontal numbers lines and located and positioned rational numbers on a coordinate plane. Students used coordinates and absolute value to find the distance between points on a number line or coordinate plane.

Current Learning

In a previous unit, students developed operations with all integers and developed algorithms for solving problems with signed numbers. Students are reinforcing adding and subtracting rational numbers including fractions and decimals. They practice addition and subtraction of integers using algorithms developed in the previous unit. Students are expected to recognize and use the additive inverse and absolute value to relate content to a real-world context. Students use prior knowledge of position on a number line to solve problems using the rules for operating with signed numbers.

Future Learning

These standards will support recognition of rational numbers versus irrational numbers in grade 8. They will also aid in the student's ability to approximate irrational numbers.

Additional Findings

In *Principles and Standards for School Mathematics,* it is stated that students should "deepen their understanding" of rational numbers and "become proficient in using them to solve problems." (p. 215)

The book also states that students in grades 6–8 "should move flexibly among fractions, decimals, and percents to order and compare rational numbers." (p. 33)

Grade 7 Mathematics, Quarter 1, Unit 1.3

Multiplying and Dividing Rational Numbers

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use the distributive property to multiply signed rational numbers.
- Understand that all quotients of integers with a non-zero divisor are rational numbers in a real-world context.
- Understand that rational numbers can be divided, provided the divisor is not zero.
- Interpret products and quotients in a real-world context.
- Apply properties of operations to multiply and divide rational numbers using appropriate strategies in real-world contexts.
- Convert rational numbers to decimal numbers using long division.

Mathematical practices to be integrated

Model with mathematics.

- Use algorithms to solve problems involving multiplication and division of rational numbers.
- Identify important quantities in a practical situation.

Use appropriate tools strategically.

- Use algorithms, paper/pencil and calculators to solve problems involving multiplication and division of rational numbers.

Attend to precision.

- Check answers to ensure that the sign associated with the answer to their problem is appropriate based on mathematical models and algorithms used.

Essential questions

- How does the distributive property of multiplication help you solve problems with rational numbers?
- Why can't you have a divisor of zero?
- What strategies can be used to multiply rational numbers?
- What strategies can be used to divide rational numbers?
- How do you know if a number is rational?
- What does the quotient of a rational number represent?
- How do you convert a rational number to a decimal?
- How can you use properties of operations as strategies to multiply and divide rational numbers?
- What are some real-world contexts that involve operations with rational numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System **7.NS**

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grades 5 and 6, students gained fluency in adding, subtracting, multiplying, and dividing fractions with like and unlike denominators. They also developed a strategy for using equivalent fractions to add and subtract fractions. Students also learned to divide multidigit positive numbers with decimals.

Current Learning

In a previous unit, students developed operations with all integers and developed algorithms for solving problems with signed numbers. Students are reinforcing multiplication and division of rational numbers. Students are practicing multiplication and division of integers using algorithms developed in a previous unit. Students recognize a rational number as a number that ends in zero or that has a repeating decimal pattern. Students are expected to recognize and use the distributive property and long division to relate content to a real-world context. Students use prior knowledge to solve problems using the rules for operating with signed numbers.

Future Learning

These standards will support recognition of rational numbers versus irrational numbers in grade 8. They will also aid in students' ability to approximate irrational numbers.

Additional Findings

In *Principles and Standards for School Mathematics,* it is stated that students should "deepen their understanding" of rational numbers and "become proficient in using them to solve problems." (p. 215)

The book also states that students in grades 6–8 "should move flexibly among fractions, decimals and percents to order and compare rational numbers." (p. 33)

According to *Curriculum Focal Points*, "Students extend their understanding of multiplication and division together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for multiplying and dividing with negative numbers make sense" (p. 37).

Grade 7 Mathematics, Quarter 1, Unit 1.4

Solving Real-Life Problems with Rational Numbers

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use numerical and algebraic expressions to solve real-life problems posed with positive and negative rational numbers in fraction form.
- Use numerical and algebraic expressions to solve real-life problems posed with positive and negative rational numbers in decimal form.
- Apply properties of operations to calculate with numbers in any form.
- Use numerical and algebraic expressions to solve real-life problems posed with positive and negative rational whole numbers.
- Convert between forms as necessary to solve real-life problems.
- Use mental math and estimation to assess reasonableness of answers.
- Use all four operations to solve real-world problems involving rational numbers.
- Apply properties of operations to evaluate expressions.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Determine which form of a solution is appropriate to the context.
- Solve a problem then check the solution using a different method.
- Make sense of their solutions.

Reason abstractly and quantitatively.

- Attend to the meaning of the quantities.
- Use different properties of operations to solve problems.

Look for and express regularity in repeated reasoning.

- Observe repeated processes and utilizing shortcuts, students continually evaluate the reasonableness of their answers.

Essential questions

- How would you represent a situation including positive and negative rational numbers using an algebraic expression?
- How would you represent a situation including positive and negative rational numbers using a numerical expression?
- When is it appropriate to write a solution as a percent?
- When is it appropriate to write a solution as a decimal?
- When is it appropriate to write a solution as a fraction?
- How do you use estimation to check the reasonableness of your answer?
- Using rational numbers, how can you solve real-world problems?
- When is it useful to use various forms of a rational numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations 7.EE

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve ~~multi-step~~ real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does

this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grades 5 and 6, students gained fluency in adding, subtracting, multiplying and dividing positive rational numbers. They also developed a strategy for using equivalent fractions to add and subtract fractions. Students learned to divide multi-digit positive numbers with decimals.

Current Learning

In a previous unit, students reinforced operations with all integers and developed algorithms for solving problems with signed numbers. Using all four mathematical operations, students are solving problems with rational numbers in all forms; including real-life applications such as, raises, sales tax, discounts, and markups. Students assess the reasonableness of their answers by using mental computation and estimation. Students are now manipulating operations including complex fractions by decomposing rational numbers into forms they are familiar to complete the operations including examples such as 3 ½ of 7 as 3 x7 + 3.5/7. Students are expected to be fluent in the four operations by the end of grade 7.

Future Learning

These standards will support recognition of rational numbers versus irrational numbers in Grade 8. They will also aid in the student's ability to approximate irrational numbers.

Additional Findings

Principles and Standards for School Mathematics states that students should be able to move flexibly within different representations of the same number, for example, "15%, 3/20, 15/100 and 15," which, depending on the context, can be used as a solution. (p. 216)

A Research Companion to Principles and Standards for School Mathematics states that teachers must not "hand over ready-made procedures, students should develop procedures themselves by reflecting on their solutions." (p. 117)